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New aspects of critical sound attenuation in magnetic systems with spin-lattice relaxation

A. Pawlak[a]

Institute of Physics, A. Mickiewicz University, Poznań, Poland

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Abstract. A general theory of critical sound propagation, including phonon-spin-energy coupling, is studied in anisotropic magnets above their transition temperature. The Kawasaki weak singularity in the ultrasonic attenuation is found as a nonasymptotic effect. A new nonasymptotic regime similar to the one in the binary mixture is also determined. The role of coupling constants and the bare relaxation times in establishing the dominance region of particular terms, is discussed.

PACS. 05.70.Jk Critical point phenomena – 62.65.+k Acoustical properties of solids

1 Introduction

The sound mode couples to a bilinear function of spin operators above the critical temperature T_C, and the attenuation coefficient is proportional to the imaginary part of a four-spin response function [1–3]. The resulting strong-singularity in the attenuation coefficient explains the experimental results in magnetic metals [4,5]. In order to interpret the observed weakly-singular or non-singular behavior of the attenuation coefficient in magnetic insulators Kawasaki [6] postulated that the interaction responsible for the critical attenuation can be considered as combining two parts. The first part was described above, the second one is a linear coupling of the longitudinal sound mode to the spin-energy density. In magnetic insulators the second contribution is believed to be dominant. A phenomenological approach gives then the sound attenuation coefficient in terms of the energy response function (the heat capacity). The latter is assumed to decay *via* spin-lattice relaxation, with the spin-lattice relaxation time showing only a weak divergence of the specific-heat type. On the other hand, the renormalization-group analysis of dynamical models of coupled spin and energy fields [7,8] showed that the singular part of the energy response function contains a term proportional to the above mentioned four-spin response function and, what is more, sufficiently close to the transition temperature T_C, the nonconserved energy relaxes with the same characteristic exponent as the nonconserved order parameter $z_E = z = 2 + c\eta$. Thus, the assumptions about the simple separability of both contributions to the ultrasonic attenuation seem to fail near T_C. Also the postulated shape of the energy response function is not correct in the asymptotic regime. Consequently, a general theory including both types of acoustic couplings as well as the interaction between energy and the order parameter would be highly desirable for a deeper understanding of the critical attenuation in magnets. In this paper, we present such a theory for an Ising-like magnet coupled to an isotropic elastic medium and an energy field. We start with a set of nonlinear Langevin equations governing the temporal behavior of nonconserved spin density, the longitudinal acoustic phonon, and nonconserved spin-energy density, which are assumed to provide a complete macroscopic description of the system near T_C. We shall limit our discussion to the disordered phase.

Performing some decoupling transformations and then extracting an irreducible with respect to phonon as well as to the energy propagators four-spin response function, we will be able to preserve also relevant nonasymptotic effects in the critical sound propagation. Our approach not only leads to a more precise expression for the acoustic self-energy, which in turn shows a novel behavior in a certain regime-resembling the critical sound attenuation in a binary liquid, but also provides considerable insight into the mechanism of ultrasonic attenuation near a critical point. It turns out that it is the ratio of bare spin-lattice and critical relaxation times, as well as their ratios to the sound frequency, which determine the dominance region of a particular sound attenuation behavior, not the relative strength of the coupling constants. On such a basis we propose a way of experimentally finding the spin-lattice relaxation time in magnetic metals. The latter quantity is of key importance in the process of Curie-point writing in magneto-optical recording.

The outline of this paper is as follows: in Section 2 the model is presented and a general expression for the acoustic self-energy is given. In Section 3 we discuss the asymptotic as well nonasymptotic regimes for the sound attenuation coefficient. The role of coupling constants and

[a] e-mail: pawlak@phys.amu.edu.pl

relaxation times is also investigated. A method for determining the spin-lattice relaxation time from the ultrasonic and hypersonic data in magnetic metals is proposed.

2 Formalism

The entropy functional for our system may be written as [7,9]

$$H = \frac{1}{2}\int d^dx \Big\{ rS^2 + (\nabla S)^2 + \tilde{u}S^4 + C_{12}\left(\sum_\alpha e_{\alpha\alpha}\right)^2 + 2C_{44}\sum_{\alpha,\beta} e_{\alpha\beta}^2 + C_S^{-1}\varepsilon^2 + g\sum_\alpha e_{\alpha\alpha}S^2 + f\varepsilon S^2 + w\varepsilon\sum_\alpha e_{\alpha\alpha}\Big\}\ , \tag{1}$$

where $S(\mathbf{x})$ denotes the one-dimensional spin variable, $\varepsilon(\mathbf{x})$ is the spin-energy density, and $e_{\alpha\beta}(\mathbf{x})$ are components of the strain tensor (they may be written in terms of phonon modes [10]). C_S is the spin heat capacity ($k_BT_C = 1$ is assumed) and $C_{\alpha\beta}$ stand for the bare elastic constants. The first three terms in (1) describe the magnetic contribution, the forth and fifth represent elastic energy in the harmonic approximation, and the sixth term comes from the lowest-order expansion of the entropy functional with respect to the energy fluctuations. The other terms describe interactions. In addition to the coupling of the longitudinal sound mode to two spin fluctuations (with the bare coupling constant g) we have taken into consideration the linear coupling of the sound mode to the energy, described by a coupling constant w. The εS^2 term is responsible for critical behavior of the specific heat [7].

The Langevin equations defining the dynamics of the model are given by [9]

$$\dot{S}_{\mathbf{k}} = -\Gamma\frac{\delta H}{\delta S_{-\mathbf{k}}} + \xi_{\mathbf{k}}, \tag{2a}$$

$$\ddot{Q}_{\mathbf{k}} = -\frac{\delta H}{\delta Q_{-\mathbf{k}}} - Dk^2\dot{Q}_{\mathbf{k}} + \eta_{\mathbf{k}}, \tag{2b}$$

$$\dot{\varepsilon} = -\gamma\frac{\delta H}{\delta \varepsilon_{-\mathbf{k}}} + \varphi_{\mathbf{k}}, \tag{2c}$$

where Q_k denotes the longitudinal phonon normal coordinate (in the isotropic model only longitudinal phonons are coupled to the spin system); $\xi_{\mathbf{k}}, \eta_{\mathbf{k}}$, and $\varphi_{\mathbf{k}}$ are the Gaussian white noises with variances related to the bare damping terms Γ, Dk^2, and γ by the Einstein relations. Only the bare spin-lattice relaxation rate γ/C_S is taken into account in the relaxation of the energy, and the heat diffusion is neglected.

Correlation and response functions of the model can be expressed most conveniently in the path-integral formalism [11,12] with a Lagrangian

$$L\left(S,\tilde{S},Q,\tilde{Q},\varepsilon,\tilde{\varepsilon}\right) = \int_\omega \sum_{\mathbf{k}} \Big\{ \Gamma\tilde{S}_{\mathbf{k},\omega}\tilde{S}_{-\mathbf{k},-\omega} + Dk^2\tilde{Q}_{\mathbf{k},\omega}\tilde{Q}_{-\mathbf{k},-\omega} + \gamma\tilde{\varepsilon}_{\mathbf{k},\omega}\tilde{\varepsilon}_{-\mathbf{k},-\omega} - \tilde{S}_{\mathbf{k},\omega}\Big(i\omega S_{-\mathbf{k},-\omega} + \Gamma\frac{\partial H}{\partial S_{\mathbf{k},\omega}}\Big) - \tilde{\varepsilon}_{\mathbf{k},\omega}\Big(i\omega\varepsilon_{-\mathbf{k},-\omega} + \gamma\frac{\partial H}{\partial \varepsilon_{\mathbf{k},\omega}}\Big) - \tilde{Q}_{\mathbf{k},\omega}\Big[(-\omega^2 + iDk^2\omega)Q_{-\mathbf{k},-\omega} + \frac{\partial H}{\partial Q_{\mathbf{k},\omega}}\Big]\Big\}, \tag{3}$$

where auxiliary response fields $\tilde{S}_{\mathbf{k},\omega}$, $\tilde{Q}_{\mathbf{k},\omega}$, and $\tilde{\varepsilon}_{\mathbf{k},\omega}$ are introduced. In (3) there is no contribution from the Jacobian as it can be set to one by defining for the step function $\Theta(0) = 0$, which excludes accusal terms in the perturbation theory [13].

Next performing the Gaussian transformations

$$\varepsilon \to \varepsilon - \mathcal{A}Q - \mathcal{B}S^2 - \mathcal{C}\tilde{Q} - \mathcal{D}\widetilde{S^2},$$
$$\tilde{\varepsilon} \to \tilde{\varepsilon} - \mathcal{E}\tilde{Q} - \mathcal{F}\widetilde{S^2}, \tag{4}$$

and

$$Q \to Q - \mathcal{G}S^2 - \mathcal{H}\widetilde{S^2},$$
$$\tilde{Q} \to \tilde{Q} - \mathcal{I}\widetilde{S^2}, \tag{5}$$

with $\mathcal{A},\mathcal{B},\mathcal{C}\ldots$ – the frequency and wave vector dependent coefficients, we are able to obtain the acoustic self-energy in the form

$$\Sigma(k,\omega) = w^2k^2D_0(\omega) + 2\left(g - wfD_0(\omega)\right)^2 k^2\langle\widetilde{S^2}_{\mathbf{k},\omega}S^2_{-\mathbf{k},-\omega}\rangle^{\overline{L}}_{irr\overline{G}_0}, \tag{6}$$

where $D_0(\omega) = (C_S^{-1} - i\omega/\gamma)^{-1}$, $\widetilde{S^2}_{\mathbf{k},\omega} = (\tilde{S}S)_{\mathbf{k},\omega}$, $S^2_{\mathbf{k},\omega} = (S^2)_{\mathbf{k},\omega}$ and the brackets denote an expectation value calculated with the effective spin Lagrangian $\overline{L}$ containing non-local strain-mediated [9,10] as well as energy-density-mediated interactions the most important of which is $\widetilde{S^2}_{\mathbf{k},\omega}S^2_{-\mathbf{k},-\omega}$ with the renormalized coupling

$$u(\omega) = \tilde{u} - 2f^2D_0(\omega) - 2\left(g - wfD_0(\omega)\right)^2 k^2\overline{G}_0(k,\omega)$$

where $\overline{G}_0^{-1}(k,\omega) = G_0^{-1} - w^2k^2D_0(\omega)$, $G_0 = (c^2k^2 - iDk^2\omega - \omega^2)^{-1}$, and $c^2 = C_{11}$ is the square of the longitudinal velocity (unitary mass density is assumed).

In (6) the four-spin response function is irreducible with respect to the phonon lines $\overline{G}_0$. We can now proceed in the standard way [10,14] replacing the strongly irrelevant parameters in $\overline{G}_0$ by zero. If we also restrict our analysis to the "weak-coupling" limit [10,15] *i.e.* we assume the couplings g and w to be small then we may replace $\langle\ldots\rangle^{\overline{L}}_{irr}$ by $\langle\ldots\rangle^{L_1}$ where the effective Lagrangian L_1 is characterized by the coupling $u_1(\omega) = \tilde{u} - 2f^2D_0(\omega) - 2\left(g - wfD_0(\omega)\right)^2 k^2\overline{G}_0(k,0)$. Someone may ask why we have not put the irrelevant coefficient γ^{-1} equal to zero,

either. However, if we did so we would obtain only asymptotic behavior characterized by the Murata exponent [1,2]. It turns out that the weak-singularity region predicted by Kawasaki can only exist in our theory as a nonasymptotic region. In order to extract this nonasymptotic behavior from (6) we must leave γ^{-1} in D_0 and note that due to $u(\omega)$ the average $\langle \dots \rangle^{L_1}$ is still reducible with respect to $D_0(\omega)$. It can be expressed in the form

$$\langle \widetilde{S^2}_{\mathbf{k},\omega} S^2_{-\mathbf{k},-\omega} \rangle^{L_1} = \frac{\langle \widetilde{S^2}_{\mathbf{k},\omega} S^2_{-\mathbf{k},-\omega} \rangle^{L_1}_{irrD_0}}{1 - 2f^2 D_0(\omega) \langle \widetilde{S^2}_{\mathbf{k},\omega} S^2_{-\mathbf{k},-\omega} \rangle^{L_1}_{irrD_0}}, \tag{7}$$

where $\langle \dots \rangle^{L_1}_{irrD_0}$ is irreducible with respect to D_0 (and with respect to $\overline{G}_0$ also). The last equation can be written as

$$\langle \widetilde{S^2}_{\mathbf{k},\omega} S^2_{-\mathbf{k},-\omega} \rangle^{L_1}_{irrD_0} = \frac{\langle \widetilde{S^2}_{\mathbf{k},\omega} S^2_{-\mathbf{k},-\omega} \rangle^{L_1}}{1 + 2f^2 D_0(\omega) \langle \widetilde{S^2}_{\mathbf{k},\omega} S^2_{-\mathbf{k},-\omega} \rangle^{L_1}}. \tag{8}$$

Because $\langle \dots \rangle^{L_1}_{irrD_0}$ is a vertex function only now we can replace γ^{-1} by zero [16] obtaining

$$\langle \widetilde{S^2}_{\mathbf{k},\omega} S^2_{-\mathbf{k},-\omega} \rangle^{L_1}_{irrD_0} = \frac{\langle \widetilde{S^2}_{\mathbf{k},\omega} S^2_{-\mathbf{k},-\omega} \rangle^{L_A}}{1 + 2f^2 C_S \langle \widetilde{S^2}_{\mathbf{k},\omega} S^2_{-\mathbf{k},-\omega} \rangle^{L_A}},$$

where L_A is the action of the model A of Halperin *et al.* [7,8] with $u_A = \tilde{u} - 2f^2 C_S - 2\tilde{g}^2 c^{-2}$ where $\tilde{g} = g - wfC_S$. The quantity $\langle \dots \rangle^{L_A}$ does not contain the irrelevant parameters and can be easily computed up to the required order [2,10,14].

Let us notice here that such a procedure is applicable only for a nonconserved energy field as for a conserved energy case (model C) D_0 contains nonstatic relevant terms and cannot be replaced by its static part in (8). Similarly $\langle \widetilde{S^2}_{\mathbf{k},\omega} S^2_{-\mathbf{k},-\omega} \rangle^{L_1}_{irrD_0}$ must be calculated with a Lagrangian L_C containing energy-generated dynamic interactions. It is worth mentioning here that the asymptotic behavior of the ultrasonic attenuation coefficient for model C was studied by Drossel and Schwabl [17]. Making use of some simplifications, adequate in this regime, they showed that there is a small difference in the asymptotic scaling functions in models A and C originating mostly from the different dynamic scaling exponents z. The discussion of nonasymptotic effects in the ultrasonic attenuation for a crossover from a nonconserved to conserved energy field will be given elsewhere and in the present paper only nonconserved energy is considered.

Inserting the last relation into (7) one obtains

$$\langle \widetilde{S^2}_{\mathbf{k},\omega} S^2_{-\mathbf{k},-\omega} \rangle^{L_1} = \frac{\langle \widetilde{S^2}_{\mathbf{k},\omega} S^2_{-\mathbf{k},-\omega} \rangle^{L_A}}{1 + 2f^2 \left(C_S - D_0(\omega)\right) \langle \widetilde{S^2}_{\mathbf{k},\omega} S^2_{-\mathbf{k},-\omega} \rangle^{L_A}},$$

and then from (6) we have

$$\Sigma = w^2 k^2 D_0 + \frac{2(g - wfD_0)^2 k^2 \langle \widetilde{S^2}_{\mathbf{k},\omega} S^2_{-\mathbf{k},-\omega} \rangle^{L_A}}{1 + 2f^2 \left(C_S - D_0\right) \langle \widetilde{S^2}_{\mathbf{k},\omega} S^2_{-\mathbf{k},-\omega} \rangle^{L_A}}. \tag{9}$$

The last equation can be transformed into the form

$$\Sigma/k^2 = \frac{2(\tilde{g}^2 - i\tilde{\omega} g^2) \langle \widetilde{S^2}_{\mathbf{k},\omega} S^2_{-\mathbf{k},-\omega} \rangle^{L_A} + w^2 C_S}{1 - i\tilde{\omega} \left(1 + v \langle \widetilde{S^2}_{\mathbf{k},\omega} S^2_{-\mathbf{k},-\omega} \rangle^{L_A}\right)}, \tag{10}$$

with $\tilde{\omega} = \omega C_S/\gamma$ and $v = 2f^2 C_S$. The ultrasonic attenuation coefficient which is proportional to the imaginary part of the acoustic self-energy can be written as

$$\frac{\alpha(\omega,t)}{\omega^2} = \frac{(\tilde{g}^2 + \tilde{\omega}^2 g^2) \mathrm{Im}\,\Psi + \tilde{\omega} v \tilde{g}^2 |\Psi|^2 - 2\tilde{\omega}\tilde{g} w f C_S \mathrm{Re}\,\Psi}{c^3 |1 - i\tilde{\omega}(1 + v\Psi)|^2}, \tag{11}$$

with the singular part of the response function $\Psi = \langle \widetilde{S^2}_{\mathbf{k},\omega} S^2_{-\mathbf{k},-\omega} \rangle^{L_A}$ obeying the scaling relation

$$\Psi = t^{-\alpha} \Phi'(y),$$

where t is proportional to the reduced temperature, $y = \omega\tau_c$ is the reduced frequency with $\tau_c = \frac{1}{\Gamma} t^{-z\nu}$ the critical relaxation time of spin fluctuations. In the ultrasonic experiments the wavelength is much longer than the correlation length ξ, so $k\xi \ll 1$ and Ψ can be evaluated for $k = 0$. The scaling function Φ' in the leading order in $\epsilon = 4 - d$ is given by [10]

$$\Phi'(y) = \left[1 + (y/2)^2\right]^{-\alpha/4\nu} \left\{ \frac{\nu}{\alpha} + \frac{i}{y} \left[i\,(1 - iy/2) \arctan(y/2) - \frac{1}{2} \ln\left(1 + (y/2)^2\right) \right] \right\} K_4,$$

where $K_4 = (8\pi^2)^{-1}$.

After neglecting the third term in the numerator of (11) (which usually can be regarded as small [18]) this equation can be written as

$$\frac{\alpha(\omega,t) c^3}{\omega^2} = \frac{W_1(\omega) t^{-(\alpha + z\nu)} \mathrm{Im}(\Phi(y)/y) + W_2 t^{-2\alpha} |\Phi(y)|^2}{|1 - ix\Phi(y)|^2}, \tag{12}$$

with $W_1(\omega) = \frac{1}{v\Gamma}(\tilde{g}^2 + \tilde{\omega}^2 g^2)$, $W_2 = \frac{\tilde{g}^2 C_S}{v\gamma_S}$, $\Phi = v\Phi'$, and an additional frequency $x = \tilde{\omega} t^{-\alpha}$ reduced with the spin-lattice relaxation time $\tau_{SL} = \frac{C_S}{\gamma_S} t^{-\alpha}$.

3 Discussion

Equation (12) behaves in three different ways depending on the relative size of t, ω and the coupling constants

as well as the bare relaxation times $\tau_{SL}^0 = C_S/\gamma_S$ and $\tau_c^0 = 1/\Gamma$. Asymptotically we obtain

$$\alpha(\omega,t) \propto \tilde{g}^2 \tau_c^0 \omega^2 t^{-(z\nu+\alpha)} \mathrm{Im}\, \Phi(y)/y \quad , \tag{13}$$

i.e. the strongly-singular behavior of the attenuation coefficient described well by Murata-Iro-Schwabl theory [1,2] in the renormalization group approach [19].

For $\tau_c^0 \ll \frac{4vK_4\nu^2}{\alpha^2}\tau_{SL}^0$ the Kawasaki type behavior [6] is regained

$$\alpha(\omega,t) \propto \tilde{g}^2 \tau_{SL}^0 \omega^2 t^{-2\alpha} \frac{|\Phi(y)|^2}{|1-ix\Phi(y)|^2}, \tag{14}$$

with the same proportionality factor $(vc^3)^{-1}$ as in (13). Note that the square of the effective coupling constant $\tilde{g}^2$ is the same in both regimes, in contrast to the naive expectations that they would be equal to g^2 and w^2, respectively. The scaling function in (14) depends on two reduced frequencies y and x but in the limit $y \longrightarrow 0$ it converts into the classical relaxation function [20]

$$\alpha(\omega,t) \propto \frac{\omega^2 C_S(t) \tau_{SL} a}{1+\omega^2 \tau_{SL}^2 a^2}, \tag{15}$$

where $C_S(t) = C_S t^{-\alpha}$ and the parameter $a = v/v^*$ ($v^* K_4 = \alpha/\nu + O(\epsilon^2)$ is the fixed point value of v) can be easily eliminated by renormalizing τ_{SL}.

One may define another interesting regime for $\tilde{\omega} \gg 1$ (assuming also $g \sim \tilde{g}$) when the second term in $W_1(\omega)$ dominates and the denominator in (12) plays a role. In this new regime one finds

$$\alpha(\omega,t) \propto g^2 \tau_c^0 \omega^2 t^{-(z\nu-\alpha)} \mathrm{Im}\, \Phi(y)^{-1}/y. \tag{16}$$

which can be obtained from the asymptotic formulae by replacing $\alpha \to -\alpha$, $\Phi \to \Phi^{-1}$ (note also the different coupling constant g instead of $\tilde{g}$). It is worth noting here that this new regime is analogous to that of the binary mixtures critical point behavior, where the coefficient of attenuation is given by [21,22,14]

$$\alpha(\omega,t) \propto \frac{-\omega \mathrm{Im}\, C_p(\omega)}{(\mathrm{Re}\, C_p)^2}\,. \tag{17}$$

with $C_p(\omega)$ as a frequency dependent specific heat at constant pressure. Applying dynamical scaling for the specific heat, the scaling law for the sound attenuation can be derived

$$\alpha(\omega,t) \propto \omega^2 t^{-\rho} F(y) \quad , \tag{18}$$

with the critical sound attenuation exponent for binary mixtures $\rho = z\nu - \alpha$. The scaling function F differs from the one in (16) mainly by a different exponent z which is about 3.05 for the binary liquid universality class [22].

It may be interesting to find the ultrasonic attenuation coefficient for vanishing coupling constant *i.e.* when $g = 0$ for the case with the phonon mode coupled only to the energy-density fluctuations. In such a case, the earlier theories predict only the Kawasaki type behavior described by (14) but we obtain

$$\frac{\alpha(\omega,t)c^3}{\omega^2} = w^2 C_S \times \frac{\tau_c^0 t^{-(\alpha+z\nu)} \mathrm{Im}(\Phi(y)/y) + \tau_{SL}^0 t^{-2\alpha} |\Phi(y)|^2}{|1-ix\Phi(y)|^2}, \tag{19}$$

where the Murata-Iro-Schwabl term is still present and can be dominant in the critical limit $\omega, t \to 0$. There is no $t^{-(z\nu-\alpha)}$ type behavior in this case.

Similarly, if the bare coupling of phonon to the energy-density fluctuation is zero we do not get only the strong singularity of a Murata-Iro-Schwabl type. Instead, that of (12) takes the form

$$\frac{\alpha(\omega,t)c^3}{\omega^2} = g^2 \frac{(1+\tilde{\omega}^2)\tau_c^0 t^{-(\alpha+z\nu)} \mathrm{Im}(\Phi(y)/y) + \tau_{SL}^0 t^{-2\alpha} |\Phi(y)|^2}{v\,|1-ix\Phi(y)|^2}, \tag{20}$$

with all three regimes present in this case.

As the vanishing of the bare coupling is not followed by that of the effective one, then the above analysis implies that it is the ratio of the bare relaxation times τ_c^0/τ_{SL}^0, and the ratio v/v^* which determine the region of dominance of a particular term in (12), rather than the relative strength of w and g. In order that the Kawasaki type behavior could be detected, there must be a frequency window for the spin-lattice bare relaxation frequency $\omega_{SL} = \gamma/C_S$

$$\omega \ll \omega_{SL}^0 \ll \omega_c^0$$

with ω the sound frequency and $\omega_c^0 = \Gamma$ the spin bare relaxation frequency. If the first inequality is obeyed then $\tilde{\omega} = \omega/\omega_{SL}^0 \ll 1$ and the "modified" Murata-Iro-Schwabl behavior (16) does not appear. On the other hand, the second inequality implies that for $t > t_{cross}$ the Kawasaki term will be dominant for a sufficiently small crossover temperature $t_{cross} = (\frac{v\tau_c^0}{4a^2\tau_{SL}^0})^{\frac{1}{z\nu-\alpha}}$.

It is evident that t_{cross} is determined mainly by the ratio of the bare relaxation times τ_{SL}^0 and τ_c^0. In magnetic insulators like $RbMnF_3$, MnF_2 and $Y_3Fe_5O_{12}$ the spin-lattice relaxation time is known from the ultrasonic attenuation experiments [23,24]. Considerable long spin-lattice relaxation time of an order of 10^{-8} s was observed near the phase transition in yttrium ferrite-garnet [24]. In MnF_2, Moran and Lűthi found $\tau_{SL} = 2.7 \times 10^{-9}$ s for $t \geq 10^{-3}$, *i.e.* it is long in comparison with the bare relaxation time for spin fluctuations which is about 10^{-11} s. Thus, the inequality $t > t_{cross}$ was obeyed in the experimental temperature range explaining the small critical sound attenuation exponent $\rho = 0.13 - 0.16$ observed in this crystal [25]. In $RbMnF_3$ a rather fast, for an insulator, spin-lattice relaxation time was found which varies from 4×10^{-10} to 2×10^{-10} s [23]. However, as determined by inelastic neutron scattering experiments [26], τ_c varied as

$0.08 \times 10^{-10} < \tau_c < 3 \times 10^{-10}$ s in the same temperature range so that $t > t_{cross}$ was also satisfied in accord with a rather small exponent $\rho = 0.27-0.32$ [25]. The experimental values of τ_{SL} for $RbMnF_3$ and MnF_2 are in qualitative agreement with the theoretical estimates obtained by Huber [27] for antiferromagnetic insulators. One thing should be pointed out here. Comparing equation (3a) in [23] with equation (15) in the present work, it is evident that the measured spin-lattice relaxation time corresponds to $a\tau_{SL}$ in the present theory. The parameter a has not been determined experimentally so far but it is obvious that it can influence the actual value of τ_{SL} obtained from the ultrasonic experiments as well as the crossover temperature.

Whereas in magnetic insulators τ_{SL} can be (at least in principle) determined experimentally from the ultrasonic data and compared with τ_c determined from inelastic neutron scattering measurements, the situation in metallic substances like rare earth metals is less favorable as large sound attenuation exponents usually observed in metals [4,25] give reason to believe that the sound attenuation is dominated by Murata-Iro-Schwabl term (13) which involves τ_c instead of τ_{SL}. There are few experimental methods allowing studies of the spin-lattice relaxation time in magnetic metals near critical point. In electron spin resonance the spin-lattice relaxation contributes, among other processes, to the line width. It was possible to assess this contribution in some metals for some temperature ranges. Extrapolating for instance the magnetic resonance data of Bloembergen [28] to the Curie temperature, one finds that $\tau_{SL} \sim 4 \times 10^{-11}$ s in nickel. Because of the lack of appropriate experimental tools, able to probe the spin-lattice energy transfer rate mediated by the spin-orbit coupling, only recently it was possible to determine the spin-lattice relaxation time in rare earth. Vaterlaus *et al.* [29] were the first to measure τ_{SL} in Gd using the pioneering technique of time resolved spin-polarized photoemission. Applying strong 10 ns laser heating pulses followed by 60 ps weak probe pulses the spin-lattice relaxation time (averaged in the temperature interval $45 < T < 225$ K) was found to be 100 ± 80 ps [29]. This result is in a very good agreement with recent theoretical estimation of τ_{SL} by Hűbner and Bennemann [30] who found $\tau_{SL} = 48$ ps for Gd. These results are also in accord with the expectations that in magnetic metals the spin-lattice relaxation times may be even a few orders of magnitude shorter than the corresponding ones in magnetic insulators and, hence, it is likely that in magnetic metals τ_{SL}^0 and τ_c^0 are of comparable value making it possible for the Murata-Iro-Schwabl term to prevail in the critical sound attenuation. However, it would be of great interest to get more experimental data on τ_{SL} as well as the value of the parameter a in magnetic metals. In a ferromagnet, τ_{SL} is also the time required to establish a new equilibrium magnetization after a sudden change of the lattice temperature. Thus, this quantity plays a crucial role not only in the theory of dynamical response of ferromagnets but it is important in the technology of magneto-optical recording as it determines, for instance, the maximum speed attainable in Curie-point writing. One way of getting round the difficulty of the lack of experimental tools able to measure the very short relaxation times, could be based on making use of the existence of the new regime (16) in the critical sound attenuation. Let us explain it in more detail. As this regime can only be detected for $\tilde{\omega} \gg 1$, and τ_{SL}^0 in magnetic metals is expected to be very short, so it is only possible to probe this regime in the nonhydrodynamic region ($y \gg 1$) where the sound attenuation saturates and the saturation value of $\alpha(\omega)$, α^{sat}, is frequency dependent:

$$\alpha^{sat} \propto \omega^{1+\alpha/z\nu} \tag{21}$$

whereas α^{sat} in the (nonhydrodynamic) Murata-Iro-Schwabl regime is described by

$$\alpha^{sat} \propto \omega^{1-\alpha/z\nu}. \tag{22}$$

It easy to show that similar relations hold also for the maximum of sound attenuation coefficient, α^{max}, which occurs in the low temperature phase and which is even more convenient to measure as it does not require precise determination of the critical temperature. The difference in the exponents is $2\alpha/z\nu \simeq 0.2$ and could be, in principle, detected in accurate measurements. The inverse of the crossover frequency, where the behavior of α^{max} changes from (22) to (21), would give then $a\tau_{SL}^0$, and from the measurements of the specific-heat one could assess the value of the parameter a. A disadvantage of the method proposed is the necessity of combining the ultrasonic attenuation measurements ($\omega < \tau_{SL}^0$) with the hypersonic data from the Brillouin scattering experiments (high frequency region), as the spin-lattice relaxation frequencies in magnetic metals lie above the ultrasonic frequency range. An undoubtable virtue of this method is the possibility of measuring the spin-lattice relaxation rate in the bulk of a crystal not only in the skin depth (in metals the optical penetration depth is 10^{-6} cm and for the microwave magnetic field in magnetic resonance it is of order $10^{-4} - 10^{-5}$ cm). Besides, the method is applicable to both the ferromagnetic as well as the antiferromagnetic materials near their transition temperature.

Finally, it is worth commenting on the Kawasaki term in the attenuation coefficient. In our approach this term is of the fourth order, whereas the other terms in (12) are of the second order, in the coupling constants. But there is no particular reason why only the limit $v/v^* \to 0$ should be considered. On the contrary, in this limit the specific heat is nonsingular (in the accessible range of temperatures), likewise the Kawasaki term in the ultrasonic attenuation. From the point of view of physics, the case with $v/v^* = O(1)$ where all terms in (12) can be treated as of the second order in small coupling constants g and w, is particularly interesting.

In our analysis we have neglected a small first-order character of the phase transition common to the compressible system [31]. Also, we would like to emphasize that the above theory is limited neither to the systems with only scalar spin nor to the magnetic phase transitions. It could also apply to a crystalline solid and other

systems – whenever the order-parameter dynamics is relaxational. However, near structural phase transitions a comparison with experiments is complicated by additional (more singular) contributions coming from the couplings of the order-parameter with the shear strains.

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